MASS TRANSFER THROUGH LAMINAR BOUNDARY LAYERS—FURTHER EXACT "SIMILAR" SOLUTIONS OF THE *b*-EQUATION

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1. INTRODUCTION

PAPER 3 of the present series [1] contains all the exact solutions to the "similar" *b*-equation available in mid-1960. Since that time a few more solutions have become available, namely those of Acrivos [2] and Sparrow and Gregg [3]. The purpose of the present note is to express these new solutions in the terms used in [1] and other papers of the series. Since the notation used is identical with that employed throughout the series, and since the present note can be regarded as an appendix to Paper 3 [1], no separate notation list will be provided here.

2. THE INTERPOLATION FORMULA OF ACRIVOS

Acrivos [2] considered the solution of the "similar" *b*-equation when the driving force B tends to -1. His resulting asymptotic expression may be written in our notation as:

$$\left(\frac{b'_0}{B}\right)_{B\to-1} = \sqrt{\left[\frac{\sigma}{(1+B)(1+\sigma)}\right]}. \tag{1}$$

Since values of (b'_0/B) for values of B close to zero are usually known, Acrivos derived and recommended an interpolation formula, valid for $-1 < B \leqslant 0$, giving (b'_0/B) in terms of the values of this quantity for $B = -1$ and $B = 0$. This formula, in the present notation, is:

$$\frac{b'_0}{B} = \left[\left(\frac{b'_0}{B}\right)_{B\to 0}^{3/2} + \left\{-B\left(\frac{b'_0}{B}\right)_{B\to-1}\right\}^{3/2}\right]^{2/3}. \tag{2}$$

We have used equation (2), together with the asymptotic formula (1) and the values of (b'_0/B) given in Paper 3 of this series [1], to construct Figs. 1, 2 and 3.

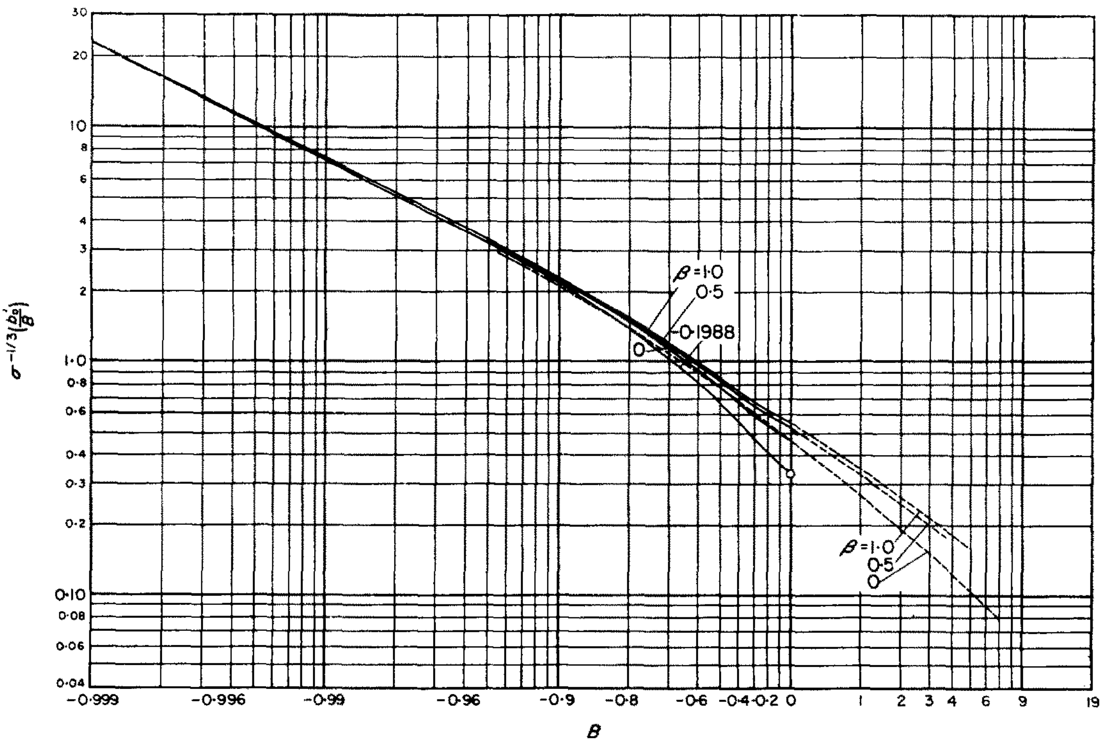

FIG. 1. $\sigma^{-1/3}(b'_0/B)$ versus B for $\sigma = 0{\cdot}7$ and various values of β. Full lines are obtained from Acrivos' interpolation formula; broken lines (or points) are exact solutions extracted from Paper 3 [1].

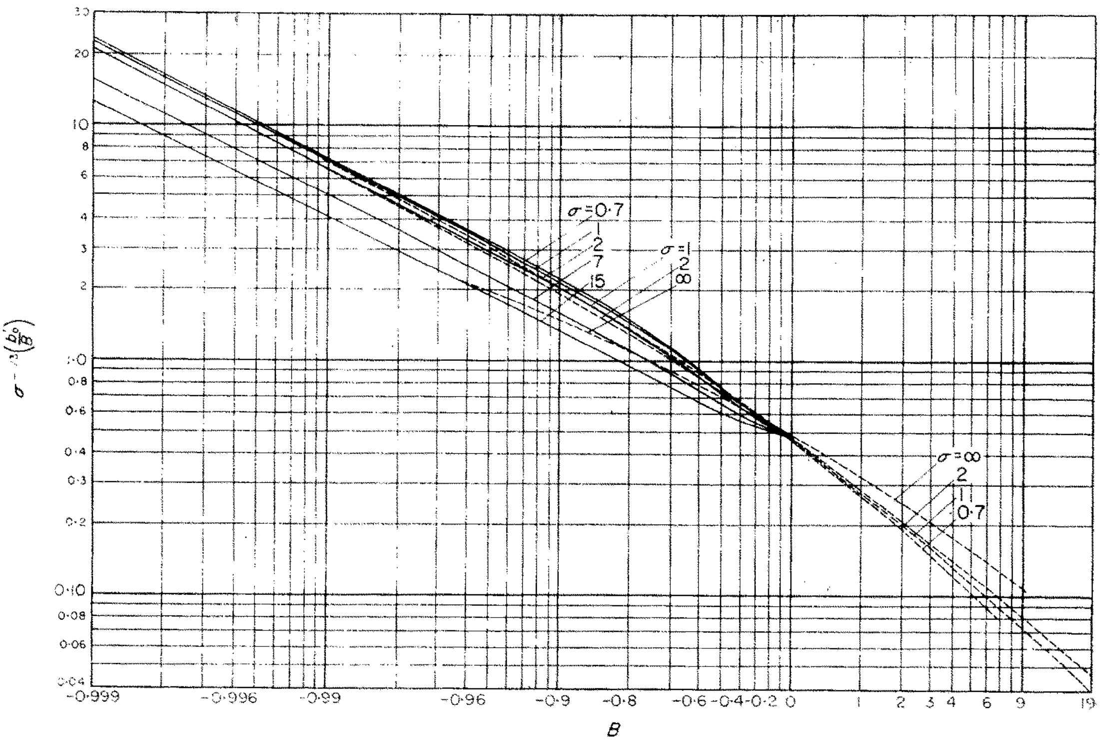

FIG. 2. $\sigma^{-1/3}(b'_0/B)$ versus B for $\beta = 0$ (the flat plate) and various values of σ. Full lines are obtained from Acrivos' interpolation formula; broken lines are exact solutions extracted from Paper 3 [1].

Figure 1 contains values of $\sigma^{-1/3}(b'_0/B)$ plotted against B for a σ-value of 0·7 and various values of β. The exact solutions extracted from Paper 3 are represented by broken lines or individual points; the full lines represent the Acrivos interpolation formula. It may be concluded that: (i) the Acrivos formula agrees well (within 5 per cent) with the exact solution for the one case in which comparison is possible ($\beta = 0$); (ii) the curves based on the Acrivos formula join fairly smoothly but exhibit an inflexion near $B = 0$ which is absent from the exact solutions; (iii) the Acrivos formula permits curves to be easily plotted in previously uncharted areas.

It should be noted that the Acrivos formula is both more accurate when B tends to -1, and easier to use, than the method presented by Spalding and Evans [1] for obtaining new solutions to the "similar" b-equation. Of course, it is restricted to negative values of B.

Figure 2 contains values of $\sigma^{-1/3}(b'_0/B)$ valid for various σ and B and for $\beta = 0$ (the flat plate). The exact solutions extracted from Paper 3 are represented by broken lines; the full lines are based on the Acrivos formula. It is seen that for σ values in the neighbourhood of 1 the agreement is still fairly good. For larger σ values, comparison with the exact solution for $\sigma = \infty$ shows that the Acrivos interpolation formula is no longer reliable. Fig. 3 contains the corresponding curves for $\beta = 1$ (plane stagnation point); it seems that similar conclusions can be drawn.

3. THE DATA OF SPARROW AND GREGG [3]

These authors considered the laminar flow on a rotating disk with a fluid of Prandtl/Schmidt number equal to 0·7. Since this aerodynamic situation has not previously been considered in the present series, it is interesting to cast the new results in the form used in the series so that similarities and differences can be perceived.

Figure 4 contains the results as a curve of $\sigma^{2/3}g/(\mu\rho\Omega)^{1/2}$ versus B. Here the only new symbol is Ω which stands for the angular velocity of the disk (radians/hour); it takes the place of $(1/\beta)(\mathrm{d}u_G/\mathrm{d}x)$ in the ordinates of Figs. 1, 2 and 3 for which it will be remembered that:

$$\sigma^{-1/3}\frac{b'_0}{B} = \sigma^{2/3}\beta^{1/2}\cdot\frac{g}{(\mu\rho \mathrm{d}u_G/\mathrm{d}x)^{1/2}} \quad (3)$$

$$= \sigma^{2/3}(2-\beta)^{1/2}\frac{g}{(\mu\rho u_G/x)^{1/2}} \quad (4)$$

Inspection of Fig. 4 shows that the curve has a form similar to that for other known solutions: the mass-transfer conductance falls with increasing B, and rises

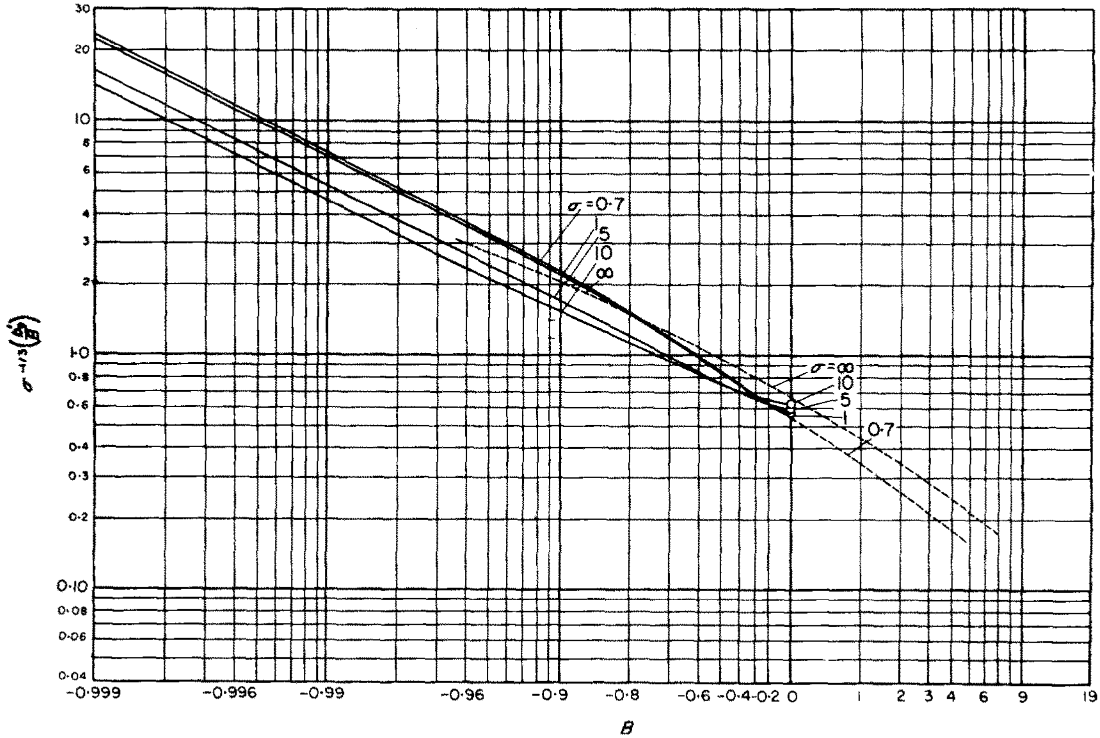

FIG. 3. $\sigma^{-1/\beta}(b'_0/B)$ versus B for $\beta = 1$ (plane stagnation point) and various values of σ. Full lines are obtained from Acrivos' interpolation formula; broken lines or individual points are exact solutions extracted from Paper 3 [1].

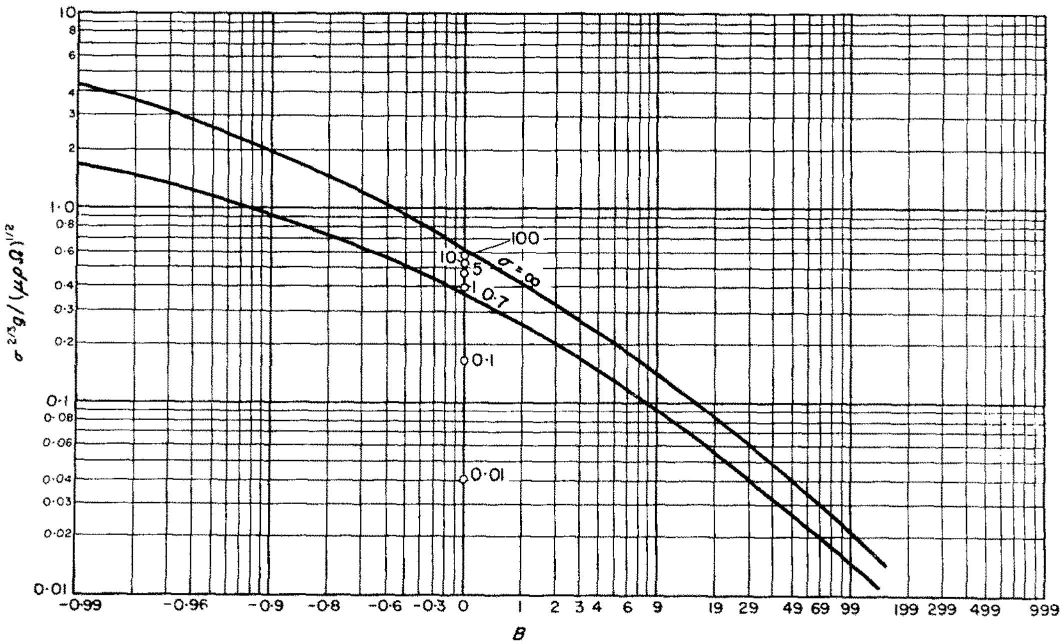

FIG. 4. $\sigma^{2/\beta}g/(\mu\rho\Omega)^{1/2}$ versus B for a rotating disk for various σ values.

with falling B, at roughly the same rate as has been found to hold for other laminar flows.

Also included on Fig. 4 is a curve valid for infinite values of σ. This case was not considered by Sparrow and Gregg, but has been solved by us, using the methods of Paper 3; it is included for comparison. It will be seen that the role of the Prandtl/Schmidt number in modifying the ordinate is similar for a rotating disk to that which it plays in the different aerodynamic flow patterns discussed in Paper 3.

A few extra points are available on the line $B = 0$; these have been deduced from solutions presented by Millsaps and Pohlhausen [4] and Sparrow and Gregg [5] as quoted by Kreith, Taylor and Chong [6].

4. OTHER NEW SOLUTIONS

Koh and Hartnett [7] have recently published solutions for negative B, $\beta = 0$, $\frac{1}{2}$ and 1, and a σ-value of 0·73. In translating these results into the present form, we have found: (i) that the necessity to read from small-scale diagrams leads to considerable uncertainty in the location of the corresponding lines on a plot such as that of Fig. 1; (ii) that the Koh–Hartnett solutions deviate systematically and considerably as $B \to -1$ from the asymptotically correct solution of Acrivos. For these reasons, we have not reproduced any of the Koh–Hartnett solutions in the present note.

The most interesting feature of the work of Koh and Hartnett is that they also obtained solutions for cases in which, though B was uniform along the surface, P_G was not.

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RADIATION AS A DIFFUSION PROCESS

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The paper "On the regularities of composite heat transfer" by Konakov, appearing in the March 1961 issue of the Journal, is in my opinion quite misleading in suggesting that combined conduction and radiation in absorbing–emitting–conducting bodies may be so treated as to obtain a simple explicit solution of the problem which is valid over the full range of variation in body dimensions—measured in mean free paths. Konakov recommends equations purporting to give the flux from hot to cold wall due to radiation and conduction acting together, for the three cases of a diathermanous medium in steady state between hot and cold parallel plates, between concentric cylinders or between concentric spheres. His recommendations for parallel walls are easy to test, since several authors have treated that case rigorously. Fig. 1 (from a lecture "Some Problems in Radiative Transport" presented by the present author at the International Heat Transfer Conference, Boulder, Colorado, August 1961) shows the Konakov recommendations, heavy lines, for comparison with the rigorous solution, light line. The graph adequately supports the generalization that radiative flux is expressible as a diffusion process $D_r(\mathrm{d}\phi/\mathrm{d}x)$ only where $\mathrm{d}\phi/\mathrm{d}x$ is constant for several mean free paths on either side of the plane of interest. Here D_r is the diffusivity of photons and ϕ is the radiation density of local space $= 4E/c$; $E = \sigma T^4$; c = velocity of light. Konakov does distinguish between molecular temperature T and radiation temperature T_r, but the equations he finally recommends do not permit the distinction. It is clear on physical grounds that when KL is small there is no dodging the solution of an integral equation or its equivalent.

The Konakov analysis also makes use of what the present author believes is an incorrect value of D_r, namely $cl_s/4$ instead of $cl_s/3$, where l_s is the mean free path of a photon or $1/K$ (K is the absorption coefficient).